Comparison of Fourier transform methods for calculating MTF

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ABSTRACT

Fourier transform methods common in infrared spectroscopy were applied to the problem of calculating the modulation transfer function (MTF) from a system's measured line spread function (LSF). Algorithms, including apodization and phase correction, are discussed in their application to remove unwanted noise from the higher frequency portion of the MTF curve. In general, these methods were found to significantly improve the calculated MTF. Apodization reduces the proportion of noise by discarding areas of the LSF where there is no appreciable signal. Phase correction significantly reduces the rectification of noise that occurs when the MTF is calculated by taking the power spectrum of the complex optical transfer function (OTF).

Keywords: MTF, FFT, modulation transfer function, Fourier transform, phase correction, apodization

1. INTRODUCTION

While there are many methods available for measuring MTF in electro-optical systems, indirect methods are among the most common. The system MTF is defined as the amplitude of the OTF, which is the Fourier transform of the line spread function (LSF). One of the most common methods used to determine the MTF is to measure the LSF either directly or by taking the derivative of the edge spread function and performing a Fourier transform on the result. Simply calculating the FFT and then calculating the amplitude by multiplying the complex OTF by its complement can introduce unwanted artifacts. Below we discuss the application of apodization and phase correction, algorithms commonly used in Fourier transform spectroscopy, to the problem of measuring MTF.

1.1 Measuring MTF

Deriving a meaningful MTF from imperfect experimental data can be challenging. Noise and detector imperfections can produce artifacts that make the extraction of a good MTF curve difficult to say the least, especially as the cutoff frequency is approached. One of the most common problems is that by taking the magnitude of the OTF any measured noise is rectified, leading to a frequency spectrum that never gets to zero at high frequencies. In many IR systems, it is difficult to maximize the signal to noise ratio without clipping the system. If the signal to noise ratio is too low, the system MTF will have additional "ringing". If the signal to noise ratio is too high, clipping will lead to a nonsensical MTF.

In modeling a sensor's performance, one of the most important quantities is the pre-sample MTF. If the measured pre-sample MTF never reaches a valid cutoff point (where it instead trends to a fixed modulation value greater than zero), then it is difficult for a system tester to determine how the actual MTF behaves. For instance, the system could cut off at the first point where the MTF approaches this floor, or this MTF floor could be included in the results until the half sample rate. In predictive models, such as NVTherm IP, these choices will greatly affect the overall calculated sensor performance.

1.2 Ideal Systems

In order to simplify the analysis and focus on the tools to be considered, much of this discussion is based on simulated MTF data. The top portion of Figure 1 shows calculated MTF curves for two systems. One is a "perfect" system with a cutoff frequency of 2.5 cyc/mR, the other is an ideal system with a 5 cyc/mR cutoff but with a ½ wave defect of defocus. The bottom portion of Figure 1 shows the same systems but with each MTF being calculated after the addition of a Gaussian noise distribution to the ESF used to calculate the MTF. The addition of the noise makes the two curves so similar as to make them difficult to distinguish, yet they represent systems with significantly different MTFs. The following discussion will demonstrate how the application of some tools commonly use in Fourier transform spectroscopy can help improve the extraction of MTF curves from a LSF with noise. It is important to note that neither of these techniques involves assumptions of a particular curve shape to the MTF. Although they have limitations, as does virtually any kind of data processing technique, they can be applied with modest care and significantly improve the data.

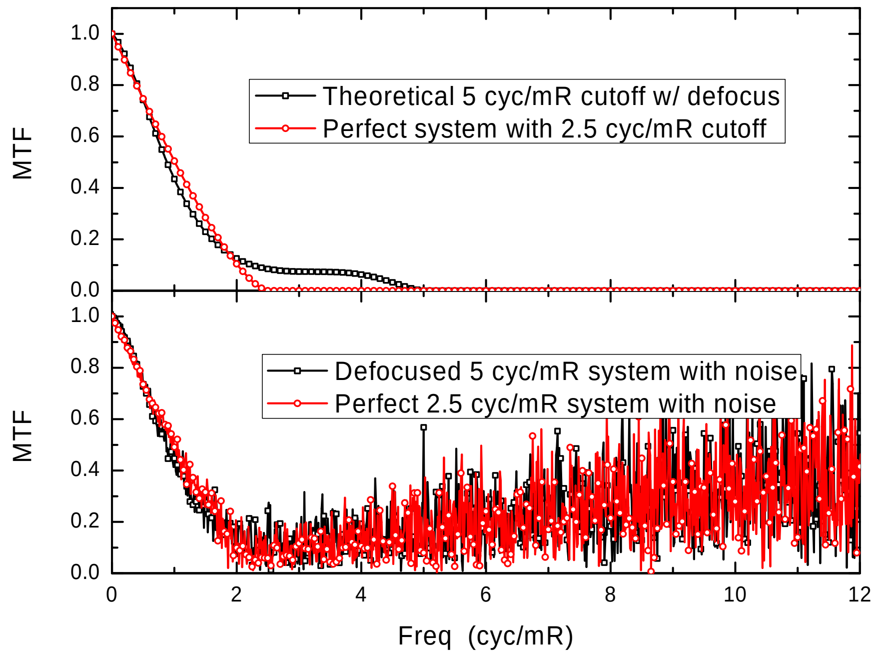

Fig. 1. The above shows plots of calculated MTF curves for two systems. The first is a system with a 5 cyc/mR cutoff, without aberrations but with a defect of focus of ½ wave. The second is an ideal system with a 2.5 cyc/mR cutoff. The upper plot shows the calculated theoretical MTF for the two systems. The lower plot shows the calculated MTF after the addition of a Gaussian noise distribution to the ESF used to derive the MTF.

2. MODELING AND SIMULATION

Previous work has been performed studying random and fixed pattern noise in MTF measurements, including comparisons of using the LSF and ESF to calculate MTF as well as the use of super-resolution to overcome aliasing present in systems where the detector under-samples the optical system response[1]. In the current study, the point of interest is in making changes to how the Fourier transform is used to extract a MTF from the LSF. The methods described below apply, regardless of how the LSF was acquired. In order to help simplify the analysis and have a known result for comparison, ideal LSF curves were generated. Noise was added with a Gaussian distribution and then the derivative was taken to generate the simulated LSF with noise.

For the ideal curve, a perfect, diffraction-limited MTF was used:

$$\hat{O}(\xi/\xi_{cutoff}) = \left(\frac{2}{\pi}\right)\left\{\arccos(\xi/\xi_{cutoff}) - \left(\xi/\xi_{cutoff}\right)\left[1-\left(\xi/\xi_{cutoff}\right)^2\right]^{1/2}\right\}, \tag{1}$$

where $\hat{O}$ is the complex optical transfer function (OTF), ξ is the spatial frequency and ξ_{cutoff} is the system cutoff frequency[2]. The system with the defect in focus was calculated using a Bessel function expansion which will not be reproduced here in order to save some space. The series and its derivation can be found in Williams and Becklund[3].

Once an OTF was calculated, an ideal LSF was generated by applying an inverse Fourier transform (3). The LSF was then numerically integrated to produce an ESF. Noise was added to the ESF in the form of a Gaussian distribution with a variance of about 0.5% of the edge maximum. This result was then numerically differentiated to produce the LSF with noise to be processed.

2.1 Fourier transform

The Fourier transform convention used was the following:

Forward transform:

$$H(f) = \int_{-\infty}^{\infty} h(x) e^{-2\pi i f x} dx \,, \tag{2}$$

and the inverse transform:

$$h(x) = \int_{-\infty}^{\infty} H(f) e^{2\pi i f x} df \,, \tag{3}$$

where $H(f)$ is the frequency response function and $h(x)$ is the spatial response function. Hence, the MTF is derived by performing the forward transform on the LSF. Both the above were implemented as discrete Fourier transform routines for use in the numerical studies.

2.2 Jargon

In the following discussion many MTF curves will be presented that were derived by several variations of calculation. In all cases the MTF is the amplitude of the OTF, though calculated or manipulated by different means. In order to keep things separate and clear for discussion, the following convention is used: If a MTF is derived by multiplying the complex OTF by its complex conjugate, it will be termed the MTF from the power spectrum or simply the power spectrum. A MTF calculated using the phase correction algorithm discussed below will be referred to as a phase corrected MTF.

3. RESULTS AND DISCUSSION

3.1 Apodization

One method used to reduce noise and help smooth a spectrum is to actually limit the LSF before it is transformed. This limiting is called apodization, the definition of which is "removing the foot." In many cases an LSF is measured out to a distance so far from the line or edge target that the system noise dominates over any contribution from the edge or slit. MTF curves rarely have any meaningful narrow features, so measuring far from the edge to get additional resolution is often not warranted. In fact, it simply adds to the proportion of noise in the MTF spectrum as $\sqrt{N}$, where N is the number of points collected. For pattern noise the situation is worse, as collecting more points simply adds coherently to that pattern's power in the MTF spectrum in direct proportion to N . The best way to limit both is by selecting the appropriate resolution of the MTF spectrum required for the intended use and then not trying to exceed it. One might be tempted to average the higher frequency MTFs to get back to the same SNR at lower resolution. This proposition would work for random noise until the point where the MTF becomes comparable to the noise. At this point rectification begins to occur, which then causes the average value to remain higher than the true MTF. Rectification causes the noise distribution to deviate from the normal distribution about zero. Because of this, the typical $\sqrt{N}$ improvement when averaging N points is not obtained. Reducing the resolution does not suffer this effect because the "noise" is removed prior to rectification. Figure 3 shows the resulting MTFs from transforming the LSF of the defocused system, shown in Figure 2, with two different lengths and averaging using the same sample spacing. The first is the full-resolution using 4096 samples, the second uses the center 1024 samples and the third is a 4 point running average of the full-resolution 4096 point result. The noise spectrum above the 5 cyc/mR cutoff for 1024 is roughly half that of the 4096 point result as would be expected. However, the 4 point moving average does not show the same improvement for the reason mentioned above.

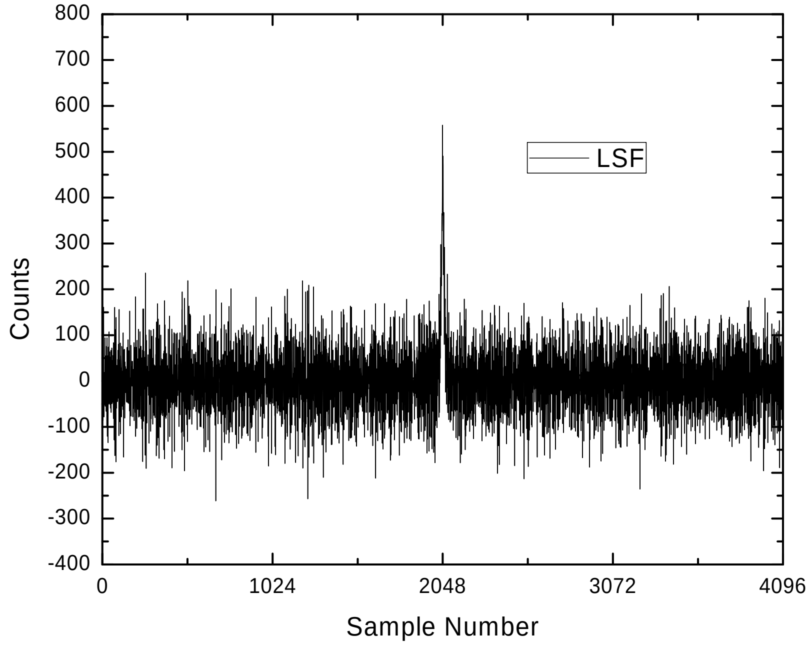

Fig. 2. The above shows the calculated LSF with noise of the 5 cyc/mR cutoff system that has a defect of focus of ½ wave. The full resolution LSF can be transformed to yield a spectrum that has resolution of 0.0125 cyc/mR. This LSF is apodized with a centered boxcar that is a total of 1024 points long with the rest of the data being filled with zeroes. This limits the effective resolution to 0.05 cyc/mR, though the calculations are interpolated to 0.0125 cyc/mR. Results for both are shown in Figures 3 and 4.

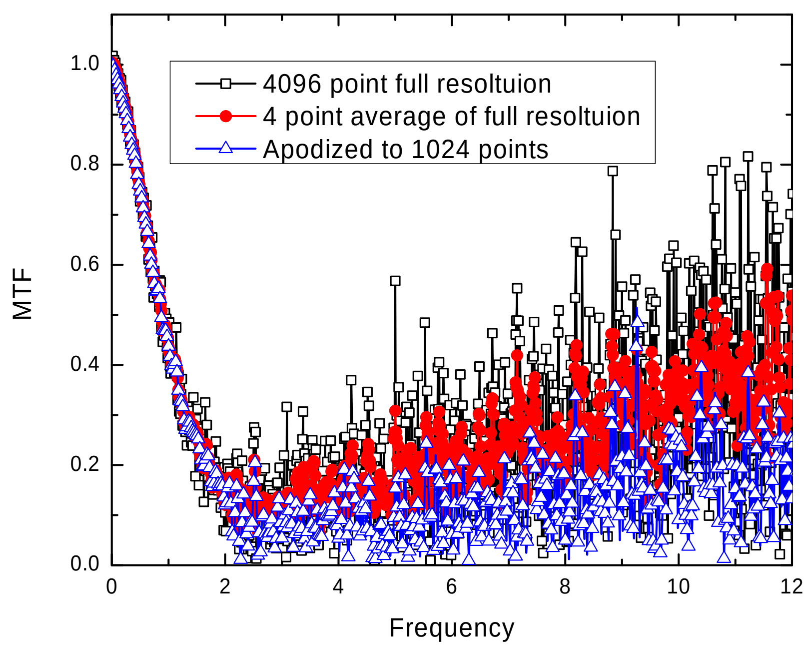

Fig. 3. This figure shows the results of calculating the MTF using the power spectrum of the LSF shown in Figure 2. This represents the defocused 5 cyc/mR system and shows both the full resolution and apodized results. Apodizing to ¼ of the full resolution reduces the high frequency noise, though it is still rectified.

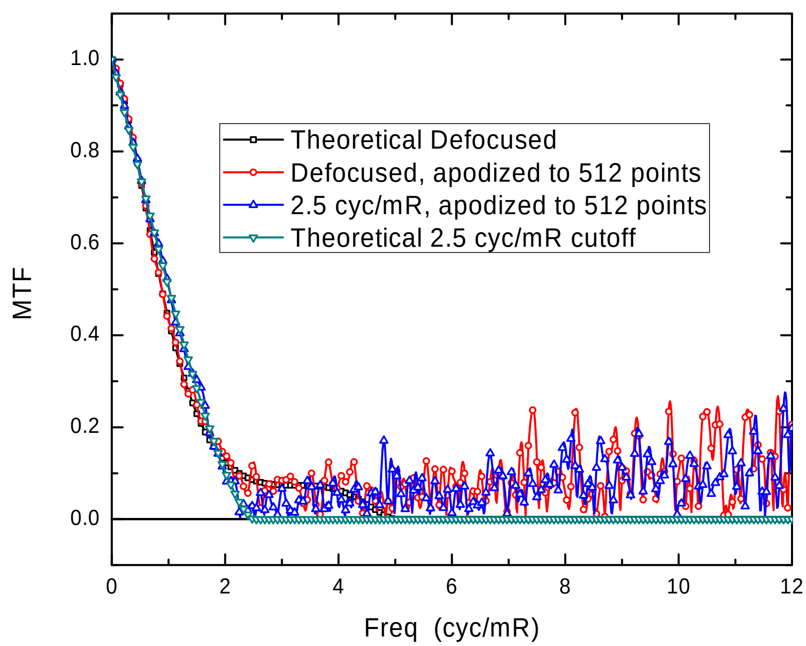

Fig. 4. This figure shows the results of the application of a 512 point apodization to both the defocused 5 cyc/mR system and the ideal 2.5 cyc/mR system. Compared to Figure 1, the apodization has helped considerably. However, the MTFs calculated using the power spectrum are still rectified and difficult to differentiate.

This choice to use a lower resolution MTF can be made after the data is collected. Even though a LSF is measured over a wide range, not all the data need be used and can be eliminated by an appropriate apodization. Apodization involves multiplying a function $f(x)$ (such as the LSF) by an apodization function $A(x)$ over the range of the function $f(x)$. The simplest apodization is the called a "boxcar":

$$A(x) = \begin{matrix} 0 & x < P1 \\ 1 & P1 \le x \le P2 \\ 0 & x > P2 \end{matrix} \qquad (4)$$

So termed because of the shape of the apodization function with which the LSF is multiplied. Other apodization functions are frequently used in FTS, however they should be used with caution as they can introduce subtle, unwanted artifacts. Figure 4 shows the result of the application of a +/- 256 point boxcar apodization to the two systems being considered. The effect is to essentially keep only the center 512 points of the LSF and replace the rest with zeroes. Because the FFT proceeds with the same number of points, the resulting MTF is also sampled at the same high resolution. However, the removal of data to be replaced by zeroes effectively reduces the real resolution resulting in the smooth curve shown. While similar at low frequencies, note that the results are not just the average of the full resolution curves at higher frequencies. Removing the points that were effectively noise has helped reduce the overall power of the noise in the spectrum.

3.2 Phase Correction

Although apodization can help, it still leaves a rectified spectrum. An algorithm exists which can help with this problem as well. Phase correction[4][5][6][7] refers to an algorithm developed to remove phase errors in Fourier transform spectroscopy (FTS). FTS involves performing a Fourier transform on an interferogram to determine a sample's response at various wavelengths of light. It is essentially the same as performing the transform of the LSF to get a system's MTF. Phase correction uses a single side (plus a little more) of a measured interferogram to calculate a spectrum that has been corrected for phase errors such as sampling error (where the center of the interferogram is not exactly sampled) as well as errors introduced by the interferometer, such as imperfect mirrors. Rather than extracting the amplitude by the performing the FFT on a two-sided LSF, it calculates the amplitude by multiplying the complex OTF by the inverse phase calculated from a low resolution LSF. Consider the following:

$$\hat{O}(f) \equiv M(f)e^{i\phi(f)} = \int_{-\infty}^{\infty} L(x)e^{-2\pi ifx}dx \tag{5}$$

where $M(f)$ is the system MTF and $L(x)$ is the LSF. If the phase function $\phi(f)$ is assumed to be slowly varying, it can be calculated using a lower resolution (apodized) version of the LSF resulting in $\phi_{lr}(f)$. Then, instead of using

$$M(f) = \left|\hat{O}(f)\right|, \tag{6}$$

use

$$M(f) = \hat{O}(f)e^{-i\phi_{lr}(f)} \tag{7}$$

to calculate the MTF. The subtle difference in the two being that by using the low resolution phase, any random high resolution noise is not rectified. Coherent noise sources such as fixed pattern noise are not improved through phase correction, though they can be reduced through judicious apodization. In optical spectroscopy it is often advantageous to collect a single-sided interferogram so that a higher resolution spectrum can be obtained without having to generate twice the required path difference. This is not usually the case in MTF measurements, in fact, since a two sided LSF is usually measured, the procedure can be performed on each side of the LSF separately and the results averaged to further reduce the noise. Should the need arise, the method works perfectly well for a single sided LSF as well, provided there is enough data on the opposite side of the center to calculate the low resolution phase.

The concept of phase correction as presented herein is fairly direct. However, its implementation does involve some attention to detail. For a full explanation of the method, see the references[4][5][6][7].

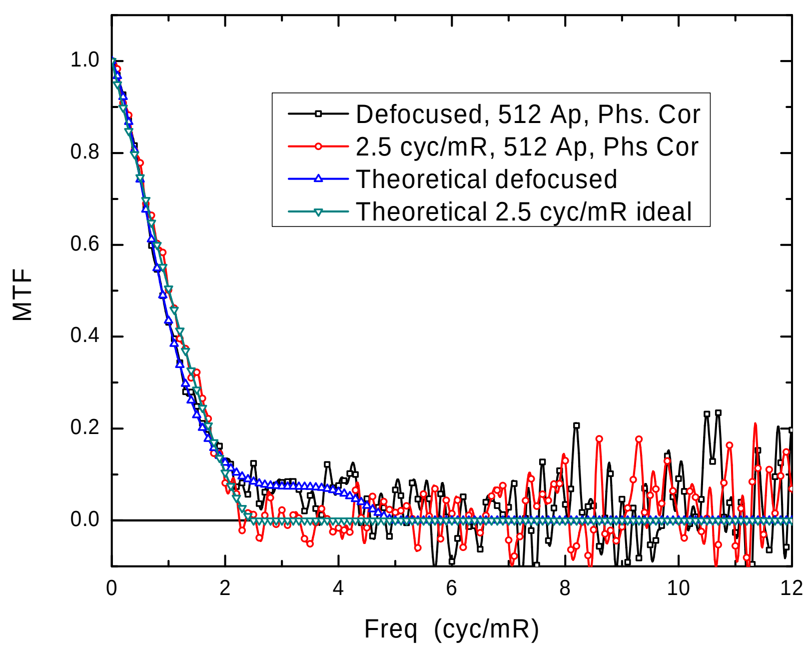

Fig. 5. This figure shows the results of the application of a 512 point apodization to both the defocused 5 cyc/mR system and the ideal 2.5 cyc/mR system as well as phase correction. Compared to Figure 4, the phase correction has helped by not rectifying the high frequency noise at low amplitudes. Despite starting with very noisy data as seen in Figure 1, the two MTF curves are now separable. Stronger apodization can smooth the curves further, but the remaining low resolution variations can become distracting, or even corrupt the low frequency MTF as seen in Figure 10.

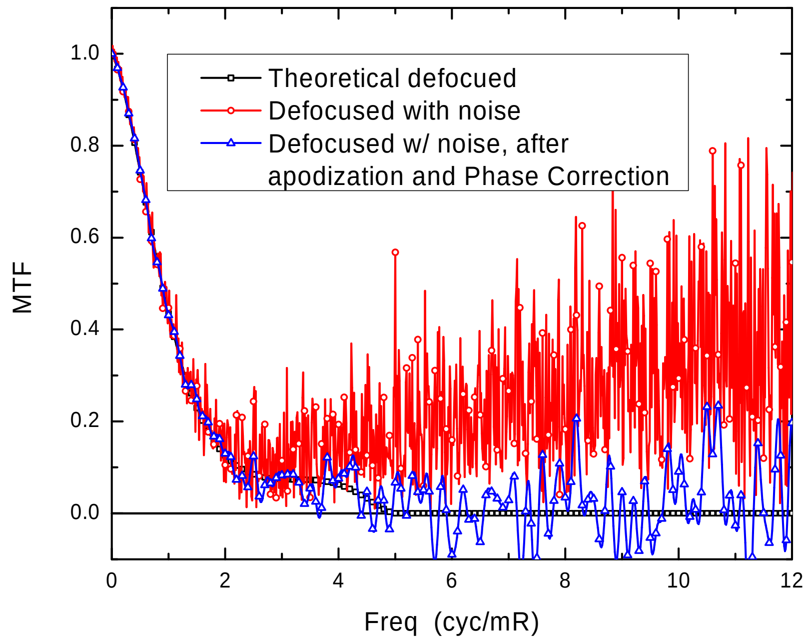

Fig. 6. This figure shows the beginning and end results for the defocused system. Note that high frequency portion of the MTF is properly moving around zero after the phase correction and that with the combination of phase correction and apodization the step out to the 5 cyc/mR is resolved.

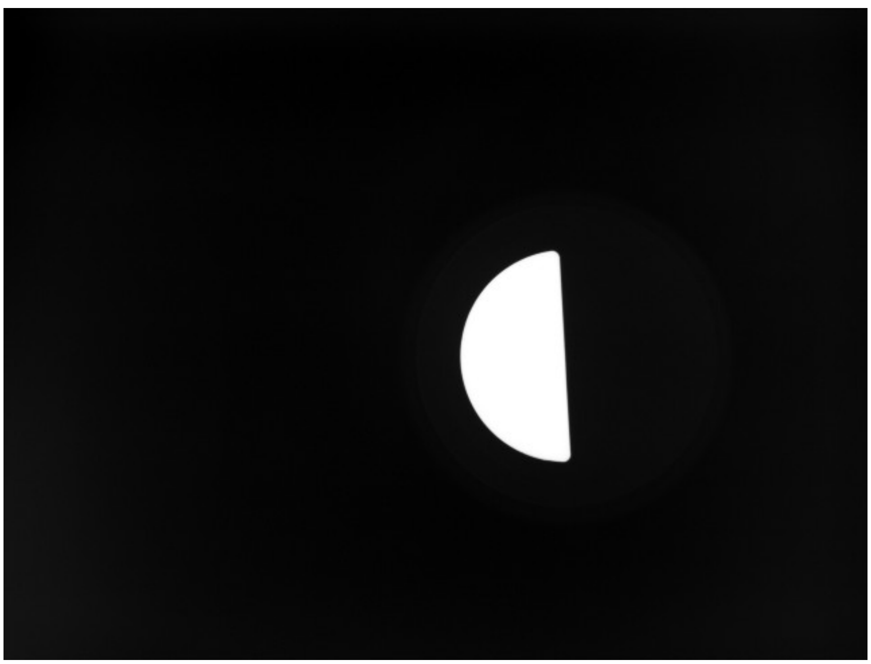

Fig. 7. This image shows the tilted edge used to derive the MTF for the real system considered here. The edge spread function was derived from the above image using a super-resolution technique.

3.3 Real systems

To demonstrate the usefulness of the procedures described above on real systems, they were applied to a real LWIR system. The system was an un-cooled VOx array with 640*480 elements with an ideal optical cutoff frequency of 7 cyc/mR and the first zero of the detector footprint MTF at 5.6 cyc/mR For this case the LSF was generated using a tilted-edge super-resolution method[1][8]. Figure 7 shows the tilted edge used to derive the LSF. Figure 8 shows the results of applying a modest apodization and phase correction to the system, compared to the full resolution, power spectrum result. As in the synthetic data, the high frequency noise is reduced and averages about zero. Using a stronger apodization makes the MTF smoother, but does not significantly reduce the average noise power. Furthermore, it has the very undesirable effect of producing distortions in the low frequency portion of the MTF as shown in Figure 9.

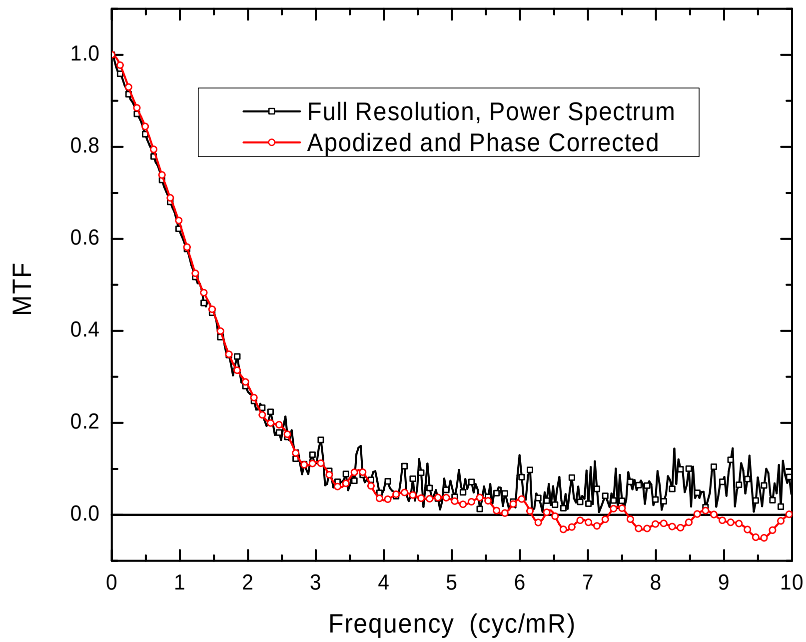

Fig. 8. This figure shows the result of applying phase correction and apodization to the real system. As with the simulated data, the high frequency noise is reduced and no longer rectified. The system has an optical cutoff frequency of 7 cyc/mR and the first zero of the detector footprint MTF is at 5.6 cyc/mR.

4. CONCLUSIONS

Fourier transform methods developed for FTS were shown to have good application in MTF measurements, especially in systems where the SNR is low. Apodization can help significantly reduce the noise contribution to a frequency spectrum if the ESF was sampled too far from the edge. Phase correction helps prevent rectification of random noise through the use of a low resolution phase calculation and a single-sided LSF. Phase correction is limited in its ability to affect coherent noise sources and apodization should not be overly strong or the risk of significant low frequency errors becomes problematic.

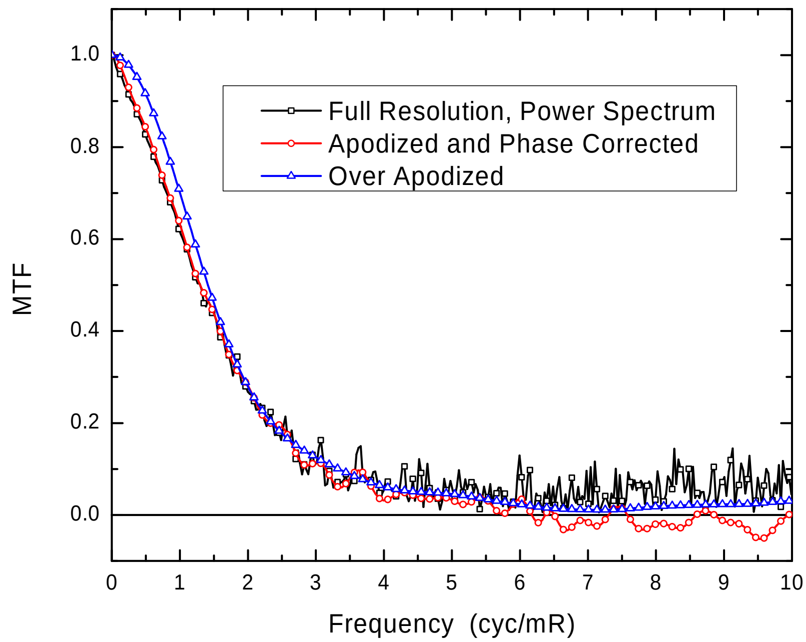

Fig. 9. This figure demonstrates the peril of trying to smooth too much through apodization. While the noise beyond the cutoff frequency is smoother with a 32x reduction in resolution, the low frequency portion of the MTF has been distorted.

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